GCE A LEVEL MARKING SCHEME

SUMMER 2025

A LEVEL
FURTHER MATHEMATICS
UNIT 4 FURTHER PURE MATHEMATICS B
1305U40-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 4 FURTHER PURE MATHEMATICS B

SUMMER 2025 MARK SCHEME

Q	Solution	Mark	Notes
1. a)	Use of formula booklet expansion for e^x with x^2: $\mathrm{e}^{x^2} = 1 + (x^2) + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \cdots$ $\mathrm{e}^{x^2} = 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \cdots$	M1 A1 [2]	
b)	Approximate $\int_0^1 \mathrm{e}^{x^2}\,\mathrm{d}x = \int_0^1 \left(1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}\right)\mathrm{d}x$ $= \left[x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42}\right]_0^1$ $= \left(1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42}\right) - 0$ $\left(= \frac{51}{35}\right) = 1.457$ correct to 3dp	M1 A1 A1 [3]	FT (a), attempt to integrate cao Final A0 for exact answer 1.463 to 3dp
		[5]	

Q	Solution	Mark	Notes
2. a)	The result is true for $n = 1$ since it gives		
	$(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos 1\theta + \mathrm{i}\sin 1\theta$.	B1	
	Let the results be true for $n = k$, ie	M1	
	$(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$		
	Consider $n = k + 1$		
	$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos\theta + \mathrm{i}\sin\theta)^k(\cos\theta + \mathrm{i}\sin\theta)$	M1	
	$= (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	A1	
	$= \cos k\theta\cos\theta - \sin k\theta\sin\theta + \mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$	A1	
	$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1	
	True for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$		
	the result is proved by mathematical induction.	A1	cso
		[7]	
b)	METHOD 1:		
	$(\cos\theta + \mathrm{i}\sin\theta)^4$		
	$= \cos^4\theta + 4\mathrm{i}\cos^3\theta\sin\theta + 6\mathrm{i}^2\cos^2\theta\sin^2\theta + 4\mathrm{i}^3\cos\theta\sin^3\theta + \mathrm{i}^4\sin^4\theta$	M1	Accept real terms only
	$= \cos^4\theta + 4\mathrm{i}\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4\mathrm{i}\cos\theta\sin^3\theta + \sin^4\theta$	A1	Accept real terms only
	$(\cos\theta + \mathrm{i}\sin\theta)^4 = \cos 4\theta + \mathrm{i}\sin 4\theta$	B1	
	Equating real parts,		
	$\therefore \cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$		
	$= (1 - \sin^2\theta)^2 - 6(1 - \sin^2\theta)\sin^2\theta + \sin^4\theta$	M1	Use of identity
	$= 1 - 2\sin^2\theta + \sin^4\theta - 6\sin^2\theta + 6\sin^4\theta + \sin^4\theta$	A1	
	$= 8\sin^4\theta - 8\sin^2\theta + 1$	A1	Convincing
	METHOD 2:		
	Use of identity $\cos 4\theta = 1 - 2\sin^2 2\theta$	(M1)	
	Use of identity $\sin 2\theta = 2\sin\theta\cos\theta$	(M1)	
	$\therefore \cos 4\theta = 1 - 2(2\sin\theta\cos\theta)^2$	(A1)	
	$= 1 - 8\sin^2\theta\cos^2\theta$	(A1)	
	$= 1 - 8\sin^2\theta\,(1 - \sin^2\theta)$	(M1)	Use of identity
	$= 1 - 8\sin^2\theta + 8\sin^4\theta$	(A1)	Convincing
		[6]	

Q	Solution	Mark	Notes
2. c)i)	METHOD 1:		Accept degrees or radian throughout
	$8\sin^4\theta - 8\sin^2\theta + 1 = 2\sin^2\theta - 1$		
	$8\sin^4\theta - 10\sin^2\theta + 2 = 0$		
	$4\sin^4\theta - 5\sin^2\theta + 1 = 0$	M1	Solvable form
	$(4\sin^2\theta - 1)(\sin^2\theta - 1) = 0$		
	Solving,	m1	
	$\sin\theta = \pm\frac{1}{2}$ or $\sin\theta = \pm 1$	A2	A1 for 2 solutions
	$\theta = 30°, 150°, 210°, 330°\ldots$ or $\theta = 90°, 270°\ldots$		Accept $\pm\theta$ values
	$\theta = 360n° + 30°$, $\theta = 360n° + 150°$, $\theta = 360n° + 210°$, $\theta = 360n° + 330°$	A1	For all 4 solns, oe
	$\theta = 360n° + 90°$, $\theta = 360n° + 270°$	A1	For both solns, oe
	METHOD 2:		
	$\cos 4\theta = 2\sin^2\theta - 1 = -\cos 2\theta$	(B1)	Identifying identity
	$\cos 4\theta + \cos 2\theta = 0$		
	Use of summation formulae,	(M1)	
	$2\cos\frac{4\theta+2\theta}{2}\cos\frac{4\theta-2\theta}{2} = 0$	(A1)	
	$2\cos 3\theta\cos\theta = 0$		
	Therefore,		
	$\cos 3\theta = 0$ or $\cos\theta = 0$	(m1)	Either
	$3\theta = 180n° + 90°$		
	$\theta = 60n° + 30°$ $\theta = 180n° + 90°$	(A2)	A1 each set
ii)	Therefore, the general solution is $\theta = 60n° + 30°$	B1	oe
		[7]	
	Radian values:		
		[20]	

$30°$	$150°$	$210°$	$330°$
$\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$
$90°$	$270°$	$180°$	$360°$
$\frac{\pi}{2}$	$\frac{3\pi}{2}$	π	2π
$60n° + 30°$		$\frac{n\pi}{3} + \frac{\pi}{6}$	

Q	Solution	Mark	Notes
3. a)	$\cosh x^2 = \dfrac{e^{x^2} + e^{-x^2}}{2}$ $\dfrac{d}{dx}(\cosh x^2)$ $= \dfrac{2xe^{x^2} - 2xe^{-x^2}}{2}$ $= 2x\left(\frac{e^{x^2} - e^{-x^2}}{2}\right)$ $= 2x \sinh x^2$	M1 A1 A1 [3]	Differentiating Convincing, answer given
b)	METHOD 1: $\int 2x^3 \sinh x^2 \, dx$ Integrating by parts with $u = x^2, \frac{dv}{dx} = 2x \sinh x^2$ $= x^2 \cosh x^2 - \int 2x \cosh x^2 \, dx$ $= x^2 \cosh x^2 - \sinh x^2 + c$	M1 m1 A1 A1	 ISW
	METHOD 2: $\int 2x^3 \sinh x^2 \, dx$ Integration by substitution with $u = x^2$ $du = 2x dx$ leading to $\int u \sinh u \, du$ Integration by parts with $u = u, \frac{dv}{dx} = \sinh u$ $= u \cosh u - \int \cosh u \, du$ $= u \cosh u - \sinh u + c$ $= x^2 \cosh x^2 - \sinh x^2 + c$	(M1) (A1) (m1) (A1) [4]	 ISW
		[7]	

Q	Solution	Mark	Notes
4. a)	$\det\mathbf{M} = (3\times 14) + (4\times -4) = 26$	B1	
	Cofactor matrix: $\begin{pmatrix} 14 & -23 & -4 \\ 0 & 13 & 0 \\ -8 & 15 & 6 \end{pmatrix}$	B2	si B1 for any 5 correct
	Adjugate matrix: $\begin{pmatrix} 14 & 0 & -8 \\ -23 & 13 & 15 \\ -4 & 0 & 6 \end{pmatrix}$	B1	FT cofactor from B1
	$\therefore \mathbf{M}^{-1} = \frac{1}{26}\begin{pmatrix} 14 & 0 & -8 \\ -23 & 13 & 15 \\ -4 & 0 & 6 \end{pmatrix}$	B1 [5]	cao No workings, no marks
b)	Multiplying matrices and use of matrix equation, leading to	M1	
	$p + 4r + 2p = -6$ $\quad(3p + 4r = -6)$ $3p - r + 2q = 11$ $\quad(3p + 2q - r = 11)$ $2p + 5r + 2r = -17$ $\quad(2p + 7r = -17)$	A2	A1 for 2 correct equations
	THEN:		
	METHOD 1: $\begin{pmatrix} 3 & 0 & 4 \\ 3 & 2 & -1 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -6 \\ 11 \\ -17 \end{pmatrix}$		
	Use of inverse from part (a):	M1	FT (a)
	$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \frac{1}{26}\begin{pmatrix} 14 & 0 & -8 \\ -23 & 13 & 15 \\ -4 & 0 & 6 \end{pmatrix}\begin{pmatrix} -6 \\ 11 \\ -17 \end{pmatrix}$	A1	
	$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$	A2	cao A1 for 2 correct solutions
	METHOD 2: $\left(\begin{array}{ccc\|c} 3 & 0 & 4 & -6 \\ 3 & 2 & -1 & 11 \\ 2 & 0 & 7 & -17 \end{array}\right)$		
	Using row reduction e.g. r1 – 1.5r3 and r2 – 1.5r3:	(M1)	
	$\left(\begin{array}{ccc\|c} 0 & 0 & -6.5 & 19.5 \\ 0 & 2 & -11.5 & 36.5 \\ 2 & 0 & 7 & -17 \end{array}\right)$	(A1)	
	Therefore, $-6.5r = 19.5$ $\quad\therefore r = -3$ $2q + (-11.5\times -3) = 36.5$ $\quad\therefore q = 1$ $2p + (7\times -3) = -17$ $\quad\therefore p = 2$	(A2)	cao A1 for 2 correct solutions
	METHOD 3: Using 1st and 3rd equations, removal of 1 variable $p = 2$ or $r = -3$ Other variable	 (M1) (A1) (A1)	 cao
	Use of second equation to find q: $(3\times 2) + 2q - (-3) = 11$ $\quad\therefore q = 1$	(A1) [7]	cao
		[12]	

Q	Solution	Mark	Notes
5.	METHOD 1:		
	Volume = $\pi\int_0^1 \sinh^2 2y\,\mathrm{d}y$	B1	Correct notation required
	Use of $\cosh 4y = 1 + 2\sinh^2 2y$	M1	M0 no workings
	$\pi\int_0^1 \left(\frac{\cosh 4y - 1}{2}\right)\mathrm{d}y$	A1	Integrable form with no more than 1 slip
	$\pi\left[\frac{1}{2}\times\frac{\sinh 4y}{4} - \frac{1}{2}y\right]_0^1$	A1	oe cao integration
	$\pi\left[\left(3.411\ldots - \frac{1}{2}\right) - (0)\right]$	m1	Substitute in correct limits
	Volume = 9.146	A1	cao
	METHOD 2:		
	Volume = $\pi\int_0^1 \sinh^2 2y\,\mathrm{d}y$	(B1)	Correct notation required
	Use of $\sinh 2y = \frac{\mathrm{e}^{2y}-\mathrm{e}^{-2y}}{2}$	(M1)	M0 no workings
	$\frac{\pi}{4}\int_0^1 (\mathrm{e}^{4y} - 2 + \mathrm{e}^{-4y})\mathrm{d}y$	(A1)	Integrable form with no more than 1 slip
	$\frac{\pi}{4}\left[\frac{\mathrm{e}^{4y}}{4} - 2y - \frac{\mathrm{e}^{-4y}}{4}\right]_0^1$	(A1)	oe cao integration
	$\frac{\pi}{4}\left[\left(\frac{\mathrm{e}^4}{4} - 2 - \frac{\mathrm{e}^{-4}}{4}\right) - \left(\frac{1}{4} - 0 - \frac{1}{4}\right)\right]$	(m1)	Substitute in correct limits
	Volume = 9.146	(A1)	cao
		[6]	If M0, award SC1 for 9.1... unsupported
		[6]	

Q	Solution	Mark	Notes
6.	Dividing both sides by $\cos x$: $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{\cos x} = \tan x$	M1	
	Integrating factor: $\mathrm{e}^{\int \frac{1}{\cos x}\mathrm{d}x} = \mathrm{e}^{\int \sec x \mathrm{d}x}$ $= \mathrm{e}^{\ln(\sec x + \tan x)} = \sec x + \tan x$	M1 A1	
	Multiplying both sides: $(\sec x + \tan x)\frac{\mathrm{d}y}{\mathrm{d}x} + y\frac{(\sec x + \tan x)}{\cos x} = (\sec x + \tan x)\tan x$	M1	
	RHS: $\sec x \tan x + \tan^2 x \rightarrow \sec x \tan x + \sec^2 x - 1$	A1	Right-side integrable form
	Integrating both sides: $y(\sec x + \tan x) = \sec x + \tan x - x + c$	m1 A1	
	Substituting $3 = 1 + 0 - 0 + c \quad \rightarrow \quad c = 2$		
	Solution: $y(\sec x + \tan x) = \sec x + \tan x - x + 2$ $y = \frac{\sec x + \tan x - x + 2}{\sec x + \tan x}$	B1	oe
		[8]	
		[8]	

Q	Solution	Mark	Notes
7. a) i)	METHOD 1: $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\sqrt{1-(2x-1)^2}} \times 2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{\sqrt{1-4x^2+4x-1}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\sqrt{x-x^2}}$	M1 A1 A1	Must be chain rule oe oe simplified
	METHOD 2: $\cos y = 2x - 1$ $-\sin y \frac{\mathrm{d}y}{\mathrm{d}x} = 2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{\sin y}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{\sqrt{1-\cos^2 y}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{\sqrt{1-(2x-1)^2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{\sqrt{1-4x^2+4x-1}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\sqrt{x-x^2}}$	(M1) (A1) (A1) [3]	oe oe simplified
ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = (3x \times \cosh 4x \times 4) + (\sinh 4x \times 3)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 12x \cosh 4x + 3 \sinh 4x$ or $3(4x \cosh 4x + \sinh 4x)$	M1 A1 A1 [3]	Product rule One term correct Fully correct and simplified
b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(\cosh x \times -\sin x) - (\cos x \times \sinh x)}{(\cosh x)^2}$ At $x = \frac{\pi}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\cosh\frac{\pi}{2}} = -0.3985 \ldots$ and $y = 0$ Gradient of normal: $\cosh\frac{\pi}{2} = \frac{-1}{-0.3985\ldots} = 2.509 \ldots$ Equation: $y - 0 = 2.509 \ldots \left(x - \frac{\pi}{2}\right)$ $y = 2.51x - 3.94$	M1 A1 B1 B1 m1 A1 [6]	Quotient rule oe Fully correct Both $\mathrm{d}y/\mathrm{d}x$ and y FT their $\mathrm{d}y/\mathrm{d}x$ Dependant on M1 Must be 2d.p.
		[12]	

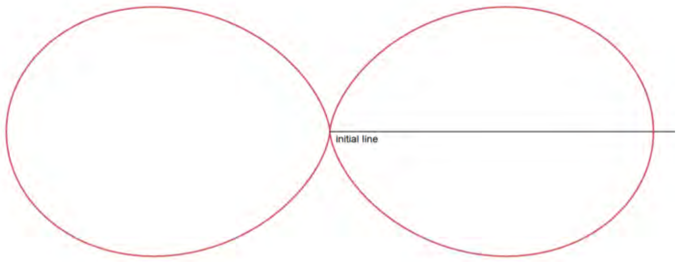
Q	Solution	Mark	Notes
8. a)	$4\cos\theta + 7\sin\theta = 4 - \tan\left(\frac{\theta}{2}\right)$		
	$4\left(\frac{1-t^2}{1+t^2}\right) + 7\left(\frac{2t}{1+t^2}\right) = 4 - t$	M1	Use of t formulae
	$4(1-t^2) + 7(2t) = (4-t)(1+t^2)$	A1	Removing fraction
	$4 - 4t^2 + 14t = 4 + 4t^2 - t - t^3$ $t^3 - 8t^2 + 15t = 0$	A1	Convincing
		[3]	
b)	$t(t^2 - 8t + 15) = 0$	B1	si
	Solving, $t = 0$ or $t^2 - 8t + 15 = 0$ $(t-3)(t-5) = 0$	M1	M0 no workings
	$t = 0$ $\quad$ $t = 3$ or $t = 5$ $\tan\left(\frac{\theta}{2}\right) = 0$ $\quad$ $\tan\left(\frac{\theta}{2}\right) = 3$ or $\tan\left(\frac{\theta}{2}\right) = 5$	A2	A1 for any two
	$\theta = 0$ $\quad$ $\theta = 2.498\ldots$ $\quad$ $\theta = 2.7468\ldots$	B2	B1 for any two Accept degrees Ignore angles outside range -1 for each additional roots in range
		[6]	
	Other factorisations could include (not exclusively) • $(t-3)(t^2-5t)$ • $(t-5)(t^2-3t)$ • *Completed squares*		
		[9]	

Q	Solution	Mark	Notes
9.	Solve auxiliary $m^2 + 16 = 0$ $m^2 = -16$ $m = \pm 4\mathrm{i}$	M1	
	Complementary function: $y = A\sin 4x + B\cos 4x$	A1	
	Use particular integral of the form $C\sin x + D\cos x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = C\cos x - D\sin x$	M1	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -C\sin x - D\cos x$	A1	
	Therefore, $-C\sin x - D\cos x + 16(C\sin x + D\cos x) = 3\sin x$	A1	Substitution
	$-C + 16C = 3 \quad \rightarrow \quad C = \frac{1}{5}$ $-D + 16D = 0 \quad \rightarrow \quad D = 0$	A1	Both values *FT C,D for M1A1M1A1*
	General Solution: $y = A\sin 4x + B\cos 4x + \frac{1}{5}\sin x$	M1	Sub and differentiate
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4A\cos 4x - 4B\sin 4x + \frac{1}{5}\cos x$	A1	
	When $x = \pi$, $y = 3$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $B = 3$ and	M1	Substituting π and one other value into either y or $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4A - \frac{1}{5} = 0 \rightarrow \quad A = \frac{1}{20}$	A1	Both A and B
	Therefore, $$y = \frac{1}{20}\sin 4x + 3\cos 4x + \frac{1}{5}\sin x$$	B1 [11]	cao
		[11]	

Q	Solution	Mark	Notes
10. a)i)	$\frac{2x^3+x^2+12}{x^3-2x^2-3x+6} = \frac{2(x^3-2x^2-3x+6)}{x^3-2x^2-3x+6} + \frac{5x^2+6x}{x^3-2x^2-3x+6}$ $\therefore \frac{2x^3+x^2+12}{x^3-2x^2-3x+6} = 2 + \frac{5x^2+6x}{x^3-2x^2-3x+6}$	M1 A1 [2]	Or polynomial division convincing
ii)	METHOD 1: $(x-2)$ is a linear factor of x^3-2x^2-3x+6 $\therefore \frac{2x^3+x^2+12}{x^3-2x^2-3x+6} = 2 + \frac{5x^2+6x}{(x-2)(x^2-3)}$ $\frac{5x^2+6x}{(x-2)(x^2-3)} = \frac{A}{x-2} + \frac{Bx+C}{x^2-3}$ $5x^2+6x = A(x^2-3) + (Bx+C)(x-2)$ When $x=2$, $32 = A$ When $x=0$, $0 = -3A - 2C \quad \rightarrow C = -48$ Comparing x^2 coefficients: $5 = A + B \quad \rightarrow B = -27$ Therefore, $\int \left(\frac{2x^3+x^2+12}{x^3-2x^2-3x+6}\right) \mathrm{d}x = \int \left(2 + \frac{32}{x-2} + \frac{-27x-48}{x^2-3}\right) \mathrm{d}x$ $= \int \left(2 + \frac{32}{x-2} + \frac{-27x}{x^2-3} + \frac{-48}{x^2-3}\right) \mathrm{d}x$ $= 2x + 32\ln\lvert x-2\rvert - \frac{27}{2}\ln\lvert x^2-3\rvert - \frac{48}{2\sqrt{3}}\ln\left\lvert\frac{x-\sqrt{3}}{x+\sqrt{3}}\right\rvert + c$ oe	B1 M1 m1 A2 m1 A3	si Partial fractions All values A1 two values Integration FT their partial fractions Fully correct ISW A2 for 4 correct terms A1 for 2 correct terms Note: $\frac{48}{2\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3}$

Q	Solution	Mark	Notes
10. a) ii)	METHOD 2: $(x-2)$ is a linear factor of x^3-2x^2-3x+6	(B1)	si
	$\therefore \frac{2x^3+x^2+12}{x^3-2x^2-3x+6} = 2 + \frac{5x^2+6x}{(x-2)(x^2-3)}$		
	$\frac{5x^2+6x}{(x-2)(x^2-3)} = \frac{A}{x-2} + \frac{B}{x+\sqrt{3}} + \frac{C}{x-\sqrt{3}}$	(M1)	Partial fractions
	$5x^2+6x = A(x+\sqrt{3})(x-\sqrt{3}) + B(x-\sqrt{3})(x-2) + C(x+\sqrt{3})(x-2)$ When $x=2$, $\quad 32 = A$ When $x=\sqrt{3}$, $\quad 15+6\sqrt{3} = C(\sqrt{3}+\sqrt{3})(\sqrt{3}-2)$ $\frac{15+6\sqrt{3}}{6-4\sqrt{3}} = C \qquad \left(= \frac{-16\sqrt{3}-27}{2}\right)$ When $x=-\sqrt{3}$, $15-6\sqrt{3} = B(-\sqrt{3}-\sqrt{3})(-\sqrt{3}-2)$ $\frac{15-6\sqrt{3}}{6+4\sqrt{3}} = B \qquad \left(= \frac{16\sqrt{3}-27}{2}\right)$	(m1) (A2)	 All values A1 two values
	Therefore, $\int \left(\frac{2x^3+x^2+12}{x^3-2x^2-3x+6}\right) \mathrm{d}x = \int \left(2 + \frac{32}{x-2} + \frac{\frac{15-6\sqrt{3}}{6+4\sqrt{3}}}{x+\sqrt{3}} + \frac{\frac{15+6\sqrt{3}}{6-4\sqrt{3}}}{x-\sqrt{3}}\right) \mathrm{d}x$	(m1)	Integration FT their partial fractions
	$= 2x + 32\ln\lvert x-2\rvert + \frac{15-6\sqrt{3}}{6+4\sqrt{3}}\ln\lvert x+\sqrt{3}\rvert + \frac{15+6\sqrt{3}}{6-4\sqrt{3}}\ln\lvert x-\sqrt{3}\rvert + c$ oe e.g. with rationalised coefficients: $= 2x + 32\ln\lvert x-2\rvert + \frac{16\sqrt{3}-27}{2}\ln\lvert x+\sqrt{3}\rvert + \frac{-16\sqrt{3}-27}{2}\ln\lvert x-\sqrt{3}\rvert + c$	(A3) [9]	Fully correct ISW A2 for 4 correct terms A1 for 2 correct terms

Q	Solution	Mark	Notes
10. b)	$4-4x-x^2=8-(x+2)^2$	B1	
	$\int_{-2}^{0}\frac{1}{\sqrt{4-4x-x^2}}\,\mathrm{d}x=\int_{-2}^{0}\frac{1}{\sqrt{8-(x+2)^2}}\,\mathrm{d}x$	M1	FT square for a^2-x^2 only
	$=\left[\sin^{-1}\left(\frac{x+2}{\sqrt{8}}\right)\right]_{-2}^{0}$	A1	
	$=\frac{\pi}{4}-0$ $=\frac{\pi}{4}$	A1	cao No workings, no marks
		[4]	
c)	METHOD 1: Mean value $=\frac{1}{2-0}\int_0^2\tanh 2x\,\mathrm{d}x$	M1	Attempt to integrate. Condone omission of ½ for M1A1
	$=\frac{1}{2}\left[\frac{\ln(\cosh 2x)}{2}\right]_0^2$	A1	Integration
	$=\frac{1}{2}(1.65359\ldots-0)$ $=0.83$	A1	cao
	METHOD 2: $\tanh 2x=\frac{\mathrm{e}^{2x}-\mathrm{e}^{-2x}}{\mathrm{e}^{2x}+\mathrm{e}^{-2x}}$ Mean value $=\frac{1}{2-0}\int_0^2\tanh 2x\,\mathrm{d}x$ $=\frac{1}{2}\int_0^2\frac{\mathrm{e}^{2x}-\mathrm{e}^{-2x}}{\mathrm{e}^{2x}+\mathrm{e}^{-2x}}\,\mathrm{d}x$	M1	Attempt to integrate. Condone omission of ½ for M1A1
	Integration by substitution with $u=\mathrm{e}^{2x}+\mathrm{e}^{-2x}$ $\mathrm{d}u=(2\mathrm{e}^{2x}-2\mathrm{e}^{-2x})\mathrm{d}x$ leading to $\frac{1}{2}\int\frac{\mathrm{d}u}{u}$ $=\frac{1}{2}\ln u$	A1	Integration
	With limits and $\frac{1}{2}$: $=\frac{1}{4}[\ln u]_{2e}^{\mathrm{e}^4+\mathrm{e}^{-4}}$ OR $=\frac{1}{4}[\ln(\mathrm{e}^{2x}+\mathrm{e}^{-2x})]_0^2$ $=\frac{1}{4}(4.000335\ldots-0.69314\ldots)$ $=0.83$	A1	cao If M0, SC1 for correct answer only
		[3]	
		[18]	

Q	Solution	Mark	Notes
11. a)	 	G2 [2]	Fully correct graph Allow G1 for • two circles • half graph
b)	$y = r\sin\theta = \cos^2\theta\sin\theta$	M1	
	Differentiating	m1	Must be product rule
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos^3\theta - 2\cos\theta\sin^2\theta$	A1	
	When parallel to initial line $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$		
	$\cos^3\theta - 2\cos\theta\sin^2\theta = 0$ $\cos\theta\,(\cos^2\theta - 2\sin^2\theta) = 0$	m1	
	$\cos\theta = 0$ (no tangent parallel to initial line)	A1	
	or $\cos^2\theta - 2\sin^2\theta = 0$ $\cos^2\theta - 2(1-\cos^2\theta) = 0$ $3\cos^2\theta - 2 = 0$		
	$\cos\theta = \pm\sqrt{\frac{2}{3}}$ $\left(\sin\theta = \pm\frac{1}{\sqrt{3}}, \tan\theta = \pm\frac{1}{\sqrt{2}}\right)$	A1	Any value
	$r = \frac{2}{3}$	A1	
	Then, $x = r\cos\theta = \frac{2}{3}\times\sqrt{\frac{2}{3}} = \frac{2\sqrt{2}}{3\sqrt{3}} = \frac{2\sqrt{6}}{9}$ $y = r\sin\theta = \frac{2}{3}\times\frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$	m1	Attempt to find x, y for any point
	Area = $2x \times 2y$	m1	
	$= 2\times\frac{2\sqrt{2}}{3\sqrt{3}}\times 2\times\frac{2}{3\sqrt{3}} = \frac{16\sqrt{2}}{27}$ oe	A1	Must be exact
		[10]	
	Alternative possible derivatives: • $y = \left(\frac{\cos 2\theta + 1}{2}\right)\sin\theta$ $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \left(\frac{\cos 2\theta+1}{2}\right)\cos\theta - \sin\theta\sin 2\theta$ • $y = \left(\frac{\cos 2\theta+1}{2}\right)\sin\theta = \frac{1}{2}\cos 2\theta\sin\theta + \frac{1}{2}\sin\theta$ $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{1}{2}\cos 2\theta\sin\theta - \sin 2\theta\sin\theta + \frac{1}{2}\cos\theta$ • $y = (1-\sin^2\theta)\sin\theta = \sin\theta - \sin^3\theta$ $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta - 3\cos\theta\sin^2\theta$		
		[12]	